

[Time:2.30 Hrs]

[ Marks:75]

Please check whether you have got the right question paper.

- N.B:
1. All question are compulsory.
  2. Figures to the right indicate full marks.
  3. Students answering in the regional language should refer in case of doubt to the main text of the paper in English.

Q.1	<p>Attempt <b><u>any three</u></b> of the following:</p> <p>a. If <math>A = \begin{bmatrix} 2 &amp; -1 \\ 1 &amp; 3 \end{bmatrix}</math>, verify Cayley Hamilton theorem and hence find <math>A^{-1}</math></p> <p>b. Solve equations using elementary transformation and find x,y and z</p> $\begin{aligned} 3x+2y+6z &= 8 \\ x+y+2z &= 6 \\ 2x+2y+5z &= 6 \end{aligned}$ <p>c. Find the inverse of following matrix by adjoint method</p> $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$ <p>d. If <math>z_1=3+2i</math> and <math>z_2=4-3i</math> then find <math>z_1+z_2</math>, <math>z_1-z_2</math>, <math>z_1.z_2</math> and <math>\frac{z_1}{z_2}</math></p> <p>e. Proof using circular and hyperbolic functions</p> <p>(i) <math>\sin ix = i \sinh x</math></p> <p>(ii) <math>\cos ix = \cosh x</math></p> <p>f. Express the following numbers in polar and exponential form</p> $\sqrt{3} + i$	15
Q.2	<p>Attempt <b><u>any three</u></b> of the following:</p> <p>a. Solve :</p> $\frac{d^3y}{dx^3} - 2 \frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$ <p>b. Solve :</p> $(D^5 - 15D^3 + 10D^2 + 60D - 72)y = 0 \quad \text{where } D = \frac{d}{dx}$ <p>c. Solve : <math>x^2 \frac{d^2y}{dx^2} + \frac{xy}{dx} = 12 \log x</math> by cauchy linear equations</p> <p>d. Solve : <math>\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 5y = 0</math></p> <p>e. Solve <math>\frac{d^4y}{dx^4} + 6 \frac{d^2y}{dx^2} + 9y = 0</math></p> <p>f. Solve : <math>x^2 \frac{d^2y}{dx^2} + \frac{xy}{dx} + y = (\log x)^2 + x \sin (\log x)</math></p>	15
Q.3	<p>a. If <math>f(t)=3</math> for <math>t \geq 0</math> then, <math>L[f(t)] = \int_0^\infty e^{-st} \cdot 3dt</math></p>	15

	<p>b. Solve : <math>L[f(t)] = \int_0^{\infty} e^{-st} \cdot f(t) dt</math></p> <p>c. Find inverse of <math>\frac{1}{s^2 - 8s + 25}</math></p> <p>d. Find <math>L^{-1} \left[ \frac{4}{s-3} \right]</math></p> <p>e. Solve : <math>f(s) = \frac{2}{s} + \frac{1}{s^2}</math></p> <p>f. Solve <math>f(t) = e^{-2t} \cdot \cos(2t)</math></p>	
Q.4	<p>a. Solve <math>\int_0^3 \int_0^2 (1 + (x-1)^2 y + 4y^2) dy dx</math></p> <p>b. Show that <math>\int_0^{\pi/2} \int_0^1 y \sin x dy dx = \frac{1}{2}</math></p> <p>c. Evaluate <math>\int_0^1 \int_0^x \int_0^{x+y} (2x + y - 1) dz dy dx</math></p> <p>d. Evaluate <math>\int_0^1 \int_2^3 xy^2 dy dx</math></p> <p>e. Evaluate <math>\int_0^{\pi/2} \int_0^5 \int_0^6 dz dr d\theta</math></p> <p>Evaluate <math>6 \int_0^1 \int_{x^2}^1 (x^2 + y) dy dx</math></p>	15
Q.5	<p>Attempt <b><u>any three</u></b> of the following:</p> <p>a. Solve: <math>\operatorname{erf}(x) + \operatorname{erfc}(x) = 1</math></p> <p>b. Solve <math>\Gamma 4.5</math></p> <p>c. Solve : <math>\operatorname{erf}(0) = 0</math></p> <p>d. Proof: <math>\operatorname{erfc}(x) + \operatorname{erfc}(-x) = 2</math></p> <p>e. Solve : <math>\beta(4.5, 3.5)</math></p> <p>f. <math>\Gamma n = 1</math> (prove gamma function property)</p>	15